5/10/16 "I make the most of all that comes and the least of all that goes."-Sara Teasdale

HW: "Geometric Series" homework section #1, 2, 3
Test 2 on Friday 5/13

AIM: How do we find the sum of a geometric series?

Warm Up:

***Exercise* #1:** Given a geometric series defined by the recursive formula $a_1 = 3$ and $a_n = a_{n-1} \cdot 2$ which of the following is the value of $S_5 = \sum_{i=1}^{5} a_i$?

previous (2)

Sum

$a_1 = 3$

$a_2 = 3(2) = 6$

$a_3 = 6(2) = 12$

$a_4 = 12(2) = 24$

$a_5 = 24(2) = 48$

$3 + 6 + 12 + 24 + 48$

$\boxed{93}$

add them

Exercise **#3:** Which of the following represents the sum of a geometric series with 8 terms whose first term is 3 and whose common ratio is 4?

~~(1) 32,756~~ ~~(3) 42,560~~

~~(2) 28,765~~ (4) 65,535

$3+12+48+192+768+3072+12288+49152$

SUM OF A FINITE GEOMETRIC SERIES

For a geometric series defined by its first term, a_1, and its common ratio, r, the sum of n terms is given by:

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{a_1 - a_1 r^n}{1-r}$$

first term — number of terms — common ratio

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$6+12+24+48+96+192+384+768$

Exercise **#4:** Find the value of the geometric series shown below. Show the calculations that lead to your final answer.

$$\underset{a_1}{6}+12+24+\cdots+\underset{a_n}{768}$$

$a_1 = 6$

$n = 8$

$r = \frac{12}{6} = 2$

Recall

$a_n = a_1 \cdot r^{n-1}$

$\frac{768}{6} = \frac{6 \cdot 2^{n-1}}{6}$

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$7 = n-1$

$8 = n$

$$\text{Sum} = \frac{6(1-2^8)}{1-2}$$

NORMAL FLOAT AUTO REAL RADIAN MP

$\frac{6(1-2^8)}{1-2}$ 1530

***Exercise* #5:** Maria places \$500 at the beginning of each year into an account that earns 5% interest compounded annually. Maria would like to determine how much money is in her account after she has made her \$500 deposit at the end of 10 years.

(a) Determine a formula for the amount, $A(t)$, that a given \$500 has grown to t-years after it was placed into this account.

$$A=500(1.05)^t$$

(b) At the end of 10 years, which will be worth more: the \$500 invested in the first year or the fourth year? Explain by showing how much each is worth at the beginning of the 11th year.

$$1^{st}=500(1.05)^{10}=\$814.45$$

$$4^{th}=500(1.05)^{6}=\$607.05$$

(c) Based on (b), write a geometric sum representing the amount of money in Maria's account after 10 years.

year 1 year 2 year 3

$$500(\underline{1.05})^{10}+500(1.05)^9+500(1.05)^8 \ldots\ldots 500(1.05)^0$$

$a_1 r^{n-1}$

(d) Evaluate the sum in (c) using the formula above.

$$\text{Sum}=\frac{a_1(1-r^n)}{1-r}=\frac{500(1-1.05^{11})}{1-1.05}=\boxed{\$7103.39}$$

***Exercise* #6:** A person places 1 penny in a piggy bank on the first day of the month, 2 pennies on the second day, 4 pennies on the third, and so on. Will this person be a millionaire at the end of a 31 day month? Show the calculations that lead to your answer.